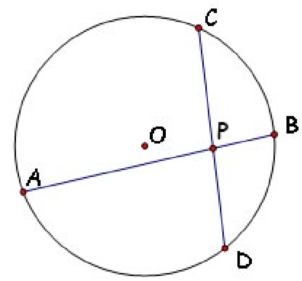
Math 5 – Trigonometry – Final Exam, Fall '12 Name___________________________
Show your work for credit. Write all responses on separate paper.

1. Show that if two chords of a circle, $\overline{AB}$ and $\overline{CD}$ intersect at P, then
   a. $\Delta APC \sim \Delta DPB$ are similar triangles. *Hint: You need to justify two congruent angles.*
   b. Use the proportionality of corresponding parts of similar triangles to show that $AP \cdot PB = DP \cdot PC$



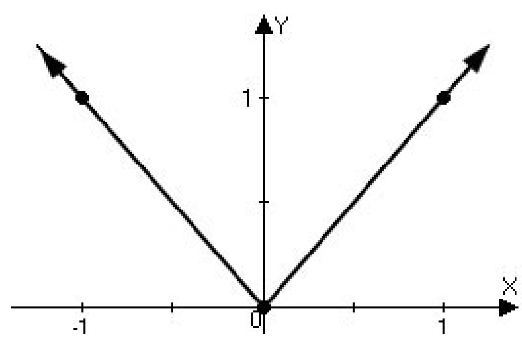
2. Find an equation for the line tangent to the circle $x^2 + y^2 = 1$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Recall that the tangent line is perpendicular to the radius line at the point of tangency.

3. Find an equation for a parabola with a vertex at (0, 12) and passing through (12, 0).

4. Consider the function $f(x) = 2x - 24$
   a. Find a formula for the inverse function, $f^{-1}(x)$.
   b. Sketch a graph for $y = f(x)$ and $y = f^{-1}(x)$ showing the coordinates of the intercepts and the symmetry through the line $y = x$.

5. Let $f(x) = |x|$, whose graph (shown at right) includes the points $(-1,1), (0,0)$ and $(1,1)$.

   Write a formula for the function that results from transforming this function by shifting down 2, reflecting across the *x*-axis and then shrinking vertically by a factor 1/2. Make a table and graph this transformed function..



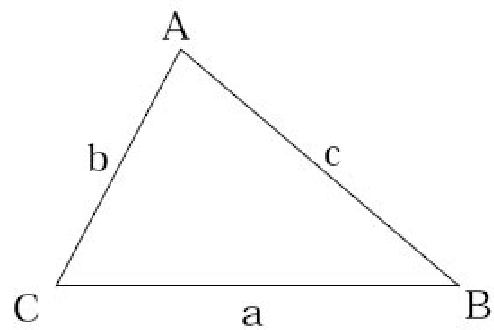
6. Consider the sinusoid $y = 12\sin\left(\frac{\pi}{12}t - \frac{\pi}{12}\right)$
   a. What is the amplitude?
   b. What is the period of oscillation?
   c. What is the phase shift?
   d. Construct a graph showing two oscillations of the sinusoid.

7. A car's wheels have radius 12 cm and rolls at a constant speed so that the tires rotate 120 revolutions per minute. How far does the car travel in 2 hours?

8. The point P is on the unit circle is in QIII and has $y = \sin(t) = -\frac{\sqrt{143}}{12}$.
   a. Find the x coordinate of P.
   b. Find $\cos(t-\pi)$
   c. Find $\sin\left(t+\frac{\pi}{2}\right)$

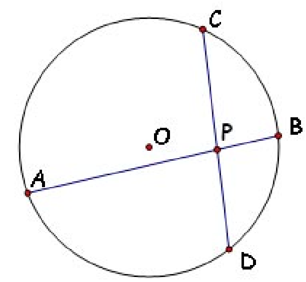
9. Approximate the interior angles of the triangle with sides of length 11, 12, and 13 to the nearest ten thousandth of a radian.

10. Consider the general triangle as shown at right.
   a. Use the formula for the area of a SAS defined triangle: $A = \frac{1}{2}xy\sin\theta$ to express the area of the triangle in three different ways.
   b. Set each of these expressions for the area equal to one another and thereby derive the law of sines.



11. Construct a large, careful graph of the conic section showing all vertices, foci and asymptotes:
   $x^2 - 4y^2 = 1$

Math 5 – Trigonometry – Final Exam Solutions Fall '12



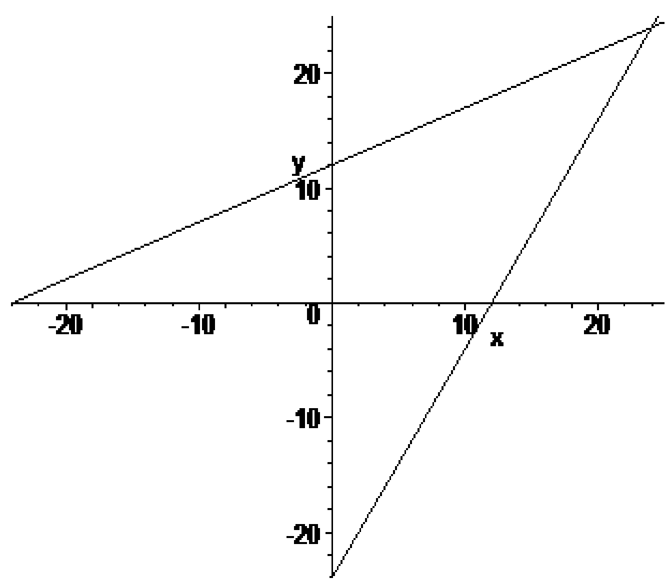
1. Show that if two chords of a circle, $\overline{AB}$ and $\overline{CD}$ intersect at *P*, then
   a. $\Delta APC \sim \Delta DPB$ are similar triangles. *Hint: You need to justify two congruent angles.*
   SOLN: $\angle APC = \angle DPB$ are vertical angles and
   $\angle ACD = \angle ABD, \angle CDA = \angle ABC$ are pairs of inscribed angles subtended by the same arcs.
   b. Use the proportionality of corresponding parts of similar triangles to show that the products are equal: $AP \cdot PB = DP \cdot PC$
   SOLN: Since corresponding parts of similar triangles are proportional,
   $\frac{AP}{DP} = \frac{PC}{PB} \Rightarrow AP \cdot PB = DP \cdot PC$

2. Find an equation for the line tangent to the circle $x^2 + y^2 = 1$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Recall that the tangent line is perpendicular to the radius line at the point of tangency.
   SOLN: The tangent line's slope is the negative reciprocal of the radius' slope: $\frac{\sqrt{3}}{2} \div \frac{1}{2} = \sqrt{3}$.
   Plugging into the point-slope equation, then: $y - \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{3}\left(x - \frac{1}{2}\right) \Leftrightarrow y = -\frac{\sqrt{3}}{3}x + \frac{2\sqrt{3}}{3}$

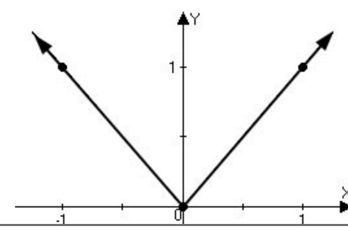
3. Find an equation for the parabola with a vertex at (0, 12) and passing through (12, 0).
   SOLN: $y - 12 = a(x-0)^2 \Leftrightarrow y = 12 + ax^2$. If this passes through (12,0), then $y = 12 + 12 + a(12)^2 \Leftrightarrow a = -\frac{1}{12}$. Therefore, $y = 12 - \frac{1}{12}x^2$.

4. Consider the function $f(x) = 2x - 24$
   a. Find a formula for the inverse function, $f^{-1}(x)$.
   SOLN: $y = 2x - 24 \Leftrightarrow x = \frac{1}{2}y + 12$
   so $f^{-1}(x) = \frac{1}{2}x + 12$

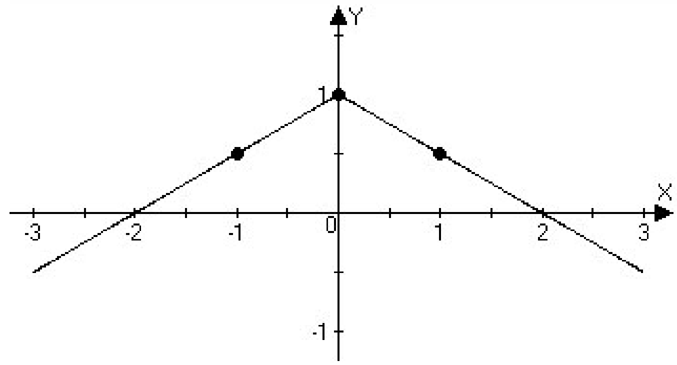
   b. Sketch a graph for $y = f(x)$ and $y = f^{-1}(x)$ showing the coordinates of the intercepts and the symmetry through the line $y = x$.
   SOLN:
   The intercepts for $f(x)$ are $(0, -24)$ and $(12,0)$. And the intercepts for $f^{-1}(x)$ are just reversed: $(-24,0)$ and $(0,12)$. These functions are graphed at right.



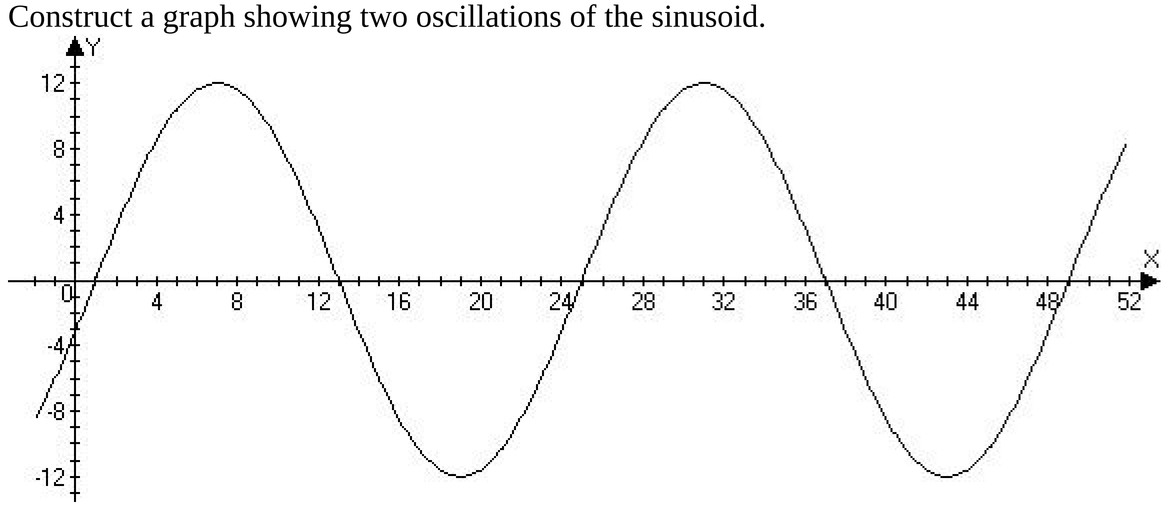
5. Let $f(x) = |x|$, whose graph (shown at right) includes the points (-1, 1), (0,0) and (1,1).



   a. Write a formula for the function that results from transforming this function by shifting down 2, reflecting across the *x*-axis and then shrinking vertically by a factor 1/2. Make a table and graph this transformed function.
   SOLN: Shift two down: $y = |x| - 2$
   Reflect across *x*-axis: $-y = |x| - 2$
   or, $y = 2 - |x|$ and
   shrink vertically by a factor of 1/2 to get
   $2y = 2 - |x|$ or, $y = 1 - \frac{1}{2}|x|$



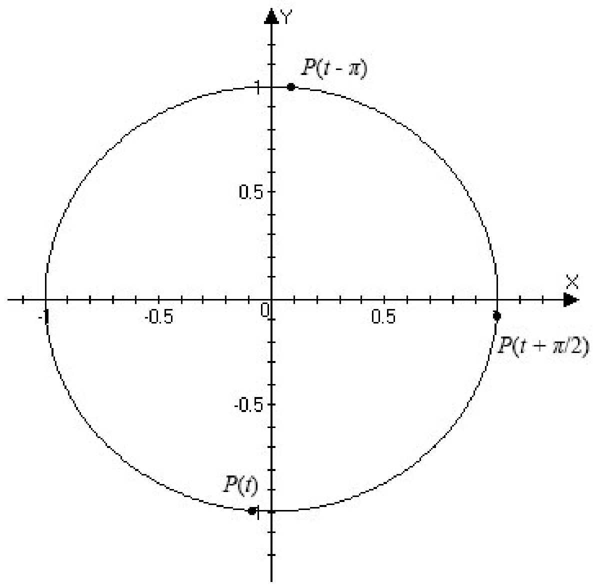
6. Consider the sinusoid $y = 12\sin\left(\frac{\pi}{12}t - \frac{\pi}{12}\right)$
   a. What is the amplitude?
   ANS: 12
   b. What is the period of oscillation?
   ANS: $\frac{2\pi}{\frac{\pi}{12}} = 24$
   c. What is the phase shift?
   ANS: 1
   d. Construct a graph showing two oscillations of the sinusoid.



7. A car's wheels have radius 12 cm and rolls at a constant speed so that the tires rotate 120 revolutions per minute. How far does the car travel in 2 hours?
   SOLN: Hello, Trigsters: the circumference of a circle is Two times Pi times R, my lord…
   $\frac{120\text{rev}}{\text{min}} \times \frac{2\pi(12\text{cm})}{\text{rev}} \times 120\text{min} = 3456\pi$ meters or about 10.86 kilometers

8. The point P is on the unit circle is in QIII and has $y = \sin(t) = -\frac{\sqrt{143}}{12}$.

a. Find the *x* coordinate of P.



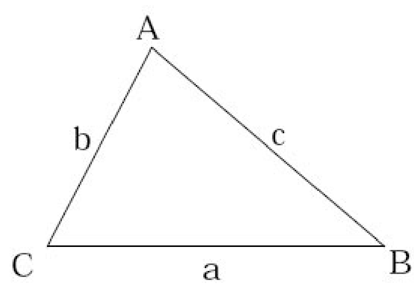
SOLN: $\frac{\sqrt{143}}{12}$ is only slightly less than 1 (12^2=144) so, as shown below, the position of this point on the unit circle is slightly to the left of $(0, -1)$.
The *x*-coordinate there is
$x = -\sqrt{1 - \frac{143}{144}} = -\frac{1}{12}$.

b. Find $\cos(t - \pi)$
SOLN: Going backwards 180° from $\left(-\frac{1}{12}, -\frac{\sqrt{143}}{12}\right)$ leads to the point $\left(\frac{1}{12}, \frac{\sqrt{143}}{12}\right)$.
So $\cos(t - \pi) = \frac{1}{12}$

c. Find $\sin\left(t + \frac{\pi}{2}\right)$
SOLN: Goving forward 90° from $\left(-\frac{1}{12}, -\frac{\sqrt{143}}{12}\right)$ leads to the point $\left(\frac{\sqrt{143}}{12}, -\frac{1}{12}\right)$
So $\sin\left(t + \frac{\pi}{2}\right) = -\frac{1}{12}$

9. Approximate the interior angles of the triangle with sides of length 11, 12, and 13 to the nearest ten thousandth of a radian.
SOLN: First use the law of cosines to find, say, the largest angle: $\theta = \cos^{-1}\left(\frac{11^2+12^2-13^2}{2\cdot 11\cdot 12}\right) = \cos^{-1}\left(\frac{4}{11}\right) \approx 1.1986 \approx 68.68°$. Now $\frac{\sin\left(\cos^{-1}\frac{4}{11}\right)}{13} = \frac{\sin\alpha}{12} \Leftrightarrow \sin\alpha = \frac{12}{13}\left(\frac{\sqrt{105}}{11}\right)$ so the angle opposite the side of length 12 is $\alpha = \sin\left(\frac{12\sqrt{105}}{143}\right) \approx 1.0350 \approx 59.30°$. This means that the angle opposite the shortest side is $\approx 180° - 68.68° - 59.30° = 52.02° \approx 0.9079$

10. Consider the general triangle as shown at right.



a. Use the formula for the area of a SAS defined triangle: $A = \frac{1}{2}xy\sin\theta$ to express the area of the triangle in three different ways.
SOLN: $A = \frac{1}{2}ab\sin\angle C = \frac{1}{2}ac\sin\angle B = \frac{1}{2}bc\sin\angle C$

b. Set each of these expressions for the area equal to one another and thereby derive the law of sines. SOLN: multiply through by 2/(*abc*).

11. Construct a large, careful graph of the conic section showing all vertices, foci and asymptotes:

$x^2 - 4y^2 = 1$

SOLN: The vertices are at (1,0) and (–1 ,0)   and the foci are at $\left(\pm\sqrt{5}/2, 0\right)$